JX-003-001544 Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

October – 2019

Statistical : Paper-S-502

(Statistical Inference)

(Old Course)

Faculty Code : 003
Subject Code : 001544

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) All questions are compulsory.

(2) Question one carry **20** marks, Questions-2 & Question-3 carry **25** marks.

(3) Students can use their own Scientific Calculator.

1 Filling the blanks and short questions : (Each **1** marks) **20**

1. A value of an estimator is called a/an __________.

2. An estimator T_n which is most concentrated about parameter θ is the _________ estimators.

3. If T_n is an estimator of a parametric function $\tau(\theta)$, the mean square error of T_n is equal to _________.

4. If an estimator T_n converges in probability to the parametric function $\tau(\theta), T_n$ is said to be a _________ estimator.

5. If T_n is an estimator of a parameter θ of the density $f(x;\theta)$ the quantity $E\left[\frac{\partial}{\partial\theta}\log f(x;\theta)\right]^2$ is called the __________.

6. If $S = s(X_1, X_2, X_3, ..., Xn)$ is a sufficient statistic for θ of density $f(x;\theta)$ and $f(x_i;\, \theta)$ for $i = 1, 2, 3, ...n$ can be factorised as $g(s,\theta)h(x)$, then $s(X_1, X_2, X_3, ...X_n)$ is a __________.

7. An estimator of $v_\theta(T_n)$ which attains lower bound for all θ is known as __________.

8. If a random sample $x_1, x_2, x_3, ...x_n$ is drawn from a population $N(\mu, \sigma^2)$, the maximum likelihood estimate of μ is __________

9. An unbiased and complete statistic is compulsorily _________.

10. Let $x_1, x_2, x_3 .., x_n$ be a random sample from a density $f(x,\theta) = \theta e^{-\theta x}$. Then the Cramer-Rao lower bound of variance of unbiased estimator is __________.

11. __________ is an unbiased estimator of p^2 in Binomial distribution.

12. Method of moments for estimating the parameters of a distribution was given by ___________ in 1894.

13. The estimation of a parameter by the method of minimum Chi-square utilizes ___________ statistic.

14. If we have a random sample of size n from a population $N(\mu, \sigma^2)$, then sample mean is __________ efficient than sample median.

15. Let there be a sample of size n from a normal population with mean μ and variance σ^2. The efficiency of median relative to the mean is ___________.

16. Define Likelihood function.

17. Name different criteria of good estimators.

18. Write likelihood function of

$$f(x,\theta) = \binom{-k}{x} \theta^k (\theta - 1)^x ; 0 \le \theta \le 1.$$

19. Write likelihood function of Poisson distribution.

20. Obtain Cramer-Rao lower bound of variance of unbiased estimator of parameter of $f(x,\theta) = xe^{-x\theta} ; 0 \le x \le \infty$.

2 (a) Write the answer any THREE : (Each **2** marks) **6**

1. Define Parameter space.
2. Define Efficiency.
3. Define Sufficiency.
4. Define Uniformly Most Powerful Test (UMP test)
5. Define ASN function of SPRT.
6. Obtain likelihood function of Negative Binomial distribution.

(b) Write the answer any THREE : (Each **3** marks) **9**

1. Obtain unbiased estimator of $\frac{kq}{p}$ of Negative Binomial distribution.

2. $\frac{\overline{x}}{n}$ is a consistent estimator of p for Binomial distribution.

3. Obtain MVUE of parameter θ for Poisson distribution. Also obtain its variance.

4. Obtain Operating Characteristic (OC) function of SPRT.

5. Give a random sample $x_1, x_2, x_3, \ldots x_n$ from distribution with p.d.f. $f(x;\theta) = \frac{1}{\theta}; 0 \le x \le \theta.$ Obtain power of the test for testing $H_0 : \theta = 1.5$ against $H_1 : \theta = 2.5$ where $c = \{x; x \ge 0.8\}$.

6. Obtain estimator of θ by method of moments in the following distribution $f(x;\theta) = \theta e^{-\theta x}$; where $0 \le x \le \infty$.

(c) Write the answer any TWO : (Each **5** marks) **10**

1. Obtain OC function for SPRT of Binomial distribution for testing $H_0 : p = p_0$ against $H_1 : p = p_1 (> p_0)$.

2. Estimate α and β in the case of Gamma distribution by the method of moments

$$f(x;\alpha,\beta) = \frac{\alpha^{\beta}}{\Gamma\beta} e^{-\alpha x} x^{\beta-1}; x \ge 0, \alpha \ge 0.$$

3. State Crammer-Rao inequality and prove it.

4. If T_1 and T_2 be two unbiased estimator of θ with variance σ_1^2, σ_2^2 and correlation ρ, What is the best unbiased linear combination of T_1 and T_2 and what is the variance of such a combination ?

5. Obtain Likelihood Ration Test :

   Let $x_1, x_2 x, x_3, ..., x_n$ random sample taken from $N(\mu, \sigma^2)$. To test $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2$.

3 (a) Write the answer any THREE : (Each **2** marks) **6**

1. Define Complete family of distribution.

2. Define Minimum Variance Bound Estimator (MVBE).

3. Define Consistency.

4. Obtain an sufficient estimator of θ by for the following distribution $f(x;\theta) = \theta^x (1-\theta)^{(1-x)}$; $x = 0,1$.

5. Obtain an unbiased estimator of θ by for the following distribution $f(x;\theta) = \frac{1}{\theta}; 0 \leq x < \theta$.

6. Show that $\sum x_i$ is a sufficient estimator of θ for Geometric distribution.

(b) Write the answer any THREE : (Each **3** marks) **9**

1. Let $x_1, x_2x, x_3, \ldots., x_n$ be random sample taken from $N(\mu, \sigma^2)$ then find sufficient estimator of μ and σ^2.

2. Obtain MLE of parameter p for the following distribution

   $f(x; p) = pq^x;\ x = 0, 1, 2, \ldots\infty$.

3. Prove that $E\left(\frac{\partial \log L}{\partial \theta}\right)^2 = -E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right)$.

4. Let p be the probability that coin will fall head in a single toss in order to test $H_0 : p\frac{1}{2}$ against $H_1 : p\frac{3}{4}$. The coin is tossed 6 times and H_0 is rejected if more than 4 head are obtained. Find the probability of type-I error, type-II error and power of test.

5. Use the Neyman Pearson lemma to obtain the best critical region for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1$ in the case of Poisson distribution with parameter λ.

6. Obtain an unbiased estimator of population mean of χ^2 distribution.

(c) Write the answer any TWO : (Each **5** marks) **10**

1. Construct SPRT of Poisson distribution for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1 (> \lambda_0)$. Also obtain OC function of SPRT.

2. Obtain MVBE of σ^2 for Normal distribution.

3. Let $x_1, x_2x, x_3, \ldots, x_n$ be random sample from the p.d.f.

$$f(x, p) = \frac{1}{(1-q^3)} \binom{3}{x} p^x q^{3-x} \quad \text{where } x = 0, 1, 2, 3$$

Estimate parameter of p by the method of moment.

4. For the double Poisson distribution

$$P(X = x) = \frac{1}{2} \frac{e^{-m_1} m_1{}^x}{x!} + \frac{1}{2} \frac{e^{-m_2} m_2{}^x}{x!} ; 0, 1, 2, \ldots$$

Show that the estimator for m_1 and m_2 by the method of moment are $\mu_1' \pm \sqrt{\mu_2' - \mu_1' - \left(\mu_1'\right)^2}$.

5. State Neyman-Pearson lemma and prove it.
